Applied Mathematics Practices for the 21st Century AMP21

The Common Core State Standards Initiative is the result of an effort at the state level coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). The most provocative aspect of the document is the Standards for Mathematical Practice. Six of the eight standards for Mathematical Practice require that students:

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1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Look for and make use of structure.

Making sense of problems and solving them is the essential characteristic of this textbook . Mathematical modeling is typically the vehicle by which OR problems are conceptualized and explored. Moreover, OR practitioners use technological tools to obtain solutions to problems and perform sensitivity analyses. Finally, sensitivity analysis, the quintessential form of explaining and interpreting results, is a clear demonstration of understanding both the nature of the problem *and* its solution.

A New Way to Learn Mathematics

We created AMP21 to develop mathematical curriculum that directly address the above standards. On the pages that follow, you will be introduced to a wide range of real world problems that operations researchers solve every day. While real world problems can be difficult to solve due to their enormous size, in many cases, their solution depends on mathematics that you have already learned in your previous high school mathematics courses. So how, you may wonder, will this course be different?

This new course in the mathematics of operations research for high school students is different because it is applications-based and problem-driven. This means that the applications of the mathematics will be upfront, not at the end of the chapter and that the key ideas will be developed within the sorts of problems that gave them birth. The applications you will see focus around making decisions in business, government, or your own personal life. In addition, mathematics is not a spectator sport. Therefore, in the text, many questions are asked but not answered. Instead, you will provide the answers. Studying mathematics in this way may require you to develop a new mindset about what mathematics is, how it can be used, and the best way to learn it.

What is Operations Research and Business Analytics?

Operations research (OR) is a scientific way to analyze problems, make decisions, and improve processes. OR professionals try to provide a sound basis for decision-making. These decisions may focus on day-to-day operations that arise in a manufacturing plant. Or, they may involve long-range issues such as designing new environmental regulations or establishing minimum prison sentence guidelines.

Operations researchers attempt to understand the structure of complex situations. They develop mathematical and computer models of a system of people, machines, and procedures. If you have ever played the game Sim City, you have manipulated a computer model. Operations researchers often use numerical, algebraic, and statistical techniques to model the decision context. Then they manipulate their models to study the behavior of the system. They use this understanding to predict how the system will behave under different rules and policies to improve system performance.

Unlike most disciplines, we can point to specific events that mark the birth of operations research. OR was born in the years just prior to World War II. The British anticipated an air war with Germany. In 1937, they began to test radar. By 1938, they were studying how to use the information radar provided to direct the operations of their fighter planes.

Until this time, the word experiment usually meant a scientist carrying out a controlled experiment in a laboratory. In contrast, this radar-fighter plane project used a multidisciplinary team of scientists. They studied actual operating conditions in the field instead of in the laboratory. They then designed experiments in the field of operations, and the new term “operations research” was born. Their goal was to understand the operation of the complete system of equipment, people, and environmental conditions (e.g. weather, nighttime). Then they tried to improve the total system’s performance. Their work was an important factor in winning the air war in Battle of Britain. OR eventually spread to all of the military services. Several of the leaders of this effort eventually won Nobel Prizes in their original fields of study.

All branches of the US Armed Forces during WWII formed similar groups of interdisciplinary scientists. These groups worked to protect naval convoys, search for enemy convoys, enhance anti-submarine warfare and improve the effectiveness of bombers. To do so, they collected data by directly observing operations. Then they built a mathematical model of the system. Next, they used the model to recommend improvements. Finally, they obtained feedback on the impact of the changes. Today, every branch of the military has its own operations research group. These OR groups include both military and civilian personnel. They play a key role in long-term strategy and weapons development. They also direct the operation of actions such as Operation Desert Storm. In addition, the National Security Agency has its own Center for Operations Research.

In the 1950s, national professional organizations were formed. These organizations published research journals and universities added OR departments. All of this raised operations research to the level of a profession. The leading professional organization is INFORMS (Institute for Operations Research and the Management Sciences). There are also operations research societies all across the globe. With regard to formal education, operations research became and remains one of the core competencies of the field of Industrial Engineering (IE). At the graduate level there is significant overlap between IE and OR. In business schools, OR generally falls within the domain of operations management or management science. Most mathematics departments also offer introductory OR courses at the junior or senior undergraduate level.

The use of OR expanded beyond the military to include other government organizations and private companies. The petroleum and chemical industries were early users of OR. They improved the performance of plants, developed natural resources and planned strategy. In the 1990s, OR models were critical enablers for multinational companies to become integrated global planners of facility operations and resource management.

In the last decade the field of operations research has gained added traction under the buzzword analytics. Numerous organizations have created analytics groups. A simple internet search yields: Google Analytics, Twitter Analytics, Pinterest Analytics, IBM Analytics, etc. Each spring INFORMS offers a Business Analytics Conference as well as special analytics conferences in areas such as healthcare.

Today, operations research and business analytics play important roles in industry and government as in:

- Airline, hospitality and entertainment industry – scheduling planes cruise ships, managing the capital investment, pricing tickets, taking reservations
- Pharmaceutical industry – managing research and development and designing sales territories;
- Delivery services – planning routes and developing pricing strategies
- Financial services – credit scoring, marketing, and internal operations
- Internet and marketing – managing the traffic to websites around the globe, tracking customers to target marketing programs
- Healthcare – hospital and healthcare clinic management, control of epidemics, effectiveness of procedures and causes of
- Commodities and lumber industry – managing mining, growing forests and cutting timber
- Local government – deploying emergency services
- Policy studies and regulation – environmental pollution, air traffic safety, AIDS, and criminal justice policy.

When Will I Ever Use This?

Volume I: Algebraic Modeling

Thomas Edwards and Kenneth Chelst

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Chapter 1: Make Hard Decisions—Multi-Criteria Decision Making (MCDM)

We all face decisions in our jobs, in our communities, and in our personal lives. For example,

- Where should a new airport, manufacturing plant, power plant, or health care clinic be located?
- Which college should I attend, or which job should I accept?
- Which car, house, computer, stereo, or health insurance plan should I buy?
- Which supplier or building contractor should I hire?

Multi-criteria decision making (MCDM) is used when one needs to make a hard decision with many criteria. The method introduced in this chapter is a structured methodology designed to handle the tradeoffs among multiple criteria. MCDM is a systematic approach to quantify an individual's preferences. Measures of interest are rescaled to numerical values on a 0–1 scale, with 0 representing the worst value of the measure and 1 representing the best. The decision maker assigns weights to each criterion to reflect the relative value of each criterion to the decision maker. This allows the direct comparison of many diverse measures. In other words, with the right tool, it really is possible to compare apples to oranges! The result of this analysis is a rank ordering of the alternatives that reflects the decision makers' preferences.

This chapter uses basic mathematics skills in an intellectually challenging environment. It was designed to provide all students with an appreciation that they can successfully apply their math skills to sophisticated decisions.

Chapter 2: Optimize Product Mix with Linear Programming (Maximization)

Chapter 2 is the introductory chapter for the optimization topics in chapters 2-7. While the succeeding chapters will extend this topic to minimization, integer programming, and binary programming, the skills in this maximization of linear programming (LP) chapter are the basic tools that will be used throughout. There is an introductory activity that involves using Lego to model the decision of making chairs and tables. This provides the students with a concrete understanding of decision variables and constraints. The problem context for this first substantive problem involves assembling two types of computers with different profit margins and labor requirements. Students are led through a graphical solution to a two decision variable problem involving two constraints. This is called the corner point principle. Lastly, the problem is expanded to include more decision variables, x_3, and x_4, to represent two additional configurations of computers. Once there are more than two decision variables, the problem cannot be solved graphically. In the next section of the text, students learn how to use SOLVER, a standard add-in to EXCEL, to solve mathematical programming problems. From this point on, Excel is a critical element of each chapter.

Chapter 3: Analyze Optimal Solutions—Sensitivity Analysis

This chapter revisits the three examples from chapter 2. In addition to solving problems, analysts are often interested in learning how sensitive their solutions are to changes in the parameters of the problem. Consider the computer assembly problem in the chapter 2. How sensitive is the solution to changes in the amount of profit that is made on each type of computer? What would be the effect of increasing the amount of available installation time or testing time? Questions such as these are part of what is called **sensitivity analysis**. This chapter is designed around the sensitivity analysis report that is part of Solver. Students learn how to explore and answer diverse what if questions.

Chapter 4: Minimize Calories or Cost with Linear Programming

This chapter presents three decision contexts in which the goal is to minimize the objective function. The first example was taken from a UN report and involves designing a nutritious diet for children in Malawi. The number of constraints and wide range of units of measures adds to the complexity of the problem. The second decision context motivates the need to use double subscripted variables to represent a pollution control decision in the Wisconsin watershed. The final decision involves more complex constraints that require students to manipulate equations to convert them to a standard form required for the Excel implementation of the model.

Chapter 5: Optimize Effectiveness or Cost with Integer Programming

In the previous chapters on linear programming, the decision variables were things that can be measured continuously such as production rates, grams of a food source, or 100-gallons of gasoline. However, in some contexts, the decision variable could be restricted. For example, a manager might need to know how many of each type of worker to hire. In this chapter, we will discuss how to solve mathematical programming problems in which the decision variables must be restricted to integer values. We will also discuss why such a restriction makes a difference in how the problem is solved and how its solution is analyzed. The chapter begins by investigating integer and non-integer solutions to linear equations in the context of purchasing advertisements to support a political campaign. The second example involves scheduling workers and supervisors at a fast food restaurant. The final decision context involves planning the shipment of truckloads of oranges to different markets in the Midwest. It illustrates the basic concepts of logistics planning. This chapter reinforces the core algebraic modeling skills of defining decision variables, framing the objective function and structuring the constraints.

Chapter 6: Optimize Selection with Binary Programming

Binary Integer Programming (BIP) problems have the same basic features as other mathematical programming problems: a set of decision variables, an objective function, a system of constraints. The distinguishing feature of BIP problems is that the possible values of the decision variables are limited to zero and one. These are called *binary decision variables*. Chapter 6 opens with a simple investment with just two decision variables and a single constraint. Students are asked to consider different values for the RHS of the constraint and investigate the implications of those changes on the optimal solution of the BIP problem. The first substantive example involves flipping houses. This example uses a complex spreadsheet to capture all of the information required to select which houses to buy and fix up.

Assignment problems are a special case of binary integer programming. Assignment problems involve matching a number of agents (e.g., athletes, students, machines) to a number of tasks (e.g., events, teams, jobs). This information is represented using matrices. If there are m agents and n tasks, then an $m \times n$ matrix of binary decision variables assigns agents to tasks. Assignment problems have a second $m \times n$ matrix, called a *cost matrix* that shows the "cost" (e.g., time, distance, monetary cost) associated with each agent performing each task.

Chapter 7: Find Optimal Locations with Algorithms

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This chapter focuses on different types of location problems. Location decisions arise in many contexts. All fast food companies, oil companies, drugstore chains or other retail outlets routinely evaluate locations for new facilities. Similar decisions are made in the public sector with regard to the location of libraries, fire stations, school buildings, and health care clinics. In some instances a simple measure of travel distance suffices to guide the decision. In other instances multiple criterion are used as in chapter 1 of this text.

The chapter starts with a simplified example involving two smoothie stands located along a single stretch of road. This example introduces the concept of minimizing the average distance traveled. Next we explore where to locate a small warehouse to store excess inventory for a downtown store. The third decision involves the location of a warehouse along a major interstate. Trucks from this warehouse are to make deliveries to different cities along the interstate. We then move to two dimensions as the decision involves locating a food stand in a downtown area. In some contexts, the preferred measure involves ensuring that all potential users of a service are within a fixed distance of the nearest facility. Any set of users that are within the prescribed distance are said to be ***covered*** by that facility. The final example utilizes binary integer programming to find the optimal locations for disaster response facilities to provide coverage of the region.

Chapter 8: Waiting in Line with Polynomials – Non-linear functions and Queues

There are relatively few broadly relevant applications of non-linear functions in high school mathematics curriculum. Mathematical models of waiting in line provide a rich array of decision contexts that utilize non-linear functions to calculate critical performance measures. Queues is the British word for waiting lines, and they are an ever present element of all societies. The chapter's first context explores how to reduce the average waiting time for people standing in line to buy tickets to a show. The second example assesses the impact of merging small rural post offices. The final example evaluates two alternative layouts of airport security screening stations. The nonlinear functions presented in this chapter include higher order polynomials as well complex exponential functions.