

Section 1.0: Introduction to Making Hard Decisions

We all face decisions in our jobs, in our communities, and in our personal lives. For example,

- Where should a new airport, manufacturing plant, power plant, or health care clinic be located?
- Which college should I attend, or which job should I accept?
- Which car, house, computer, stereo, or health insurance plan should I buy?
- Which supplier or building contractor should I hire?

Decisions such as these involve comparing alternatives that have strengths or weaknesses with regard to multiple objectives of interest to the decision. For example, your criteria in buying health insurance might be to minimize cost *and* maximize protection. Sometimes these multiple criteria get in each other's way.

Multi-criteria decision making (MCDM) is used when one needs to make a hard decision with many criteria. In this chapter, you will see one form of multi-criteria decision making. The method introduced in this chapter is a structured methodology designed to handle the tradeoffs among multiple criteria.

A Little History

One of the first applications of this method of MCDM involved the study of possible locations for a new airport in Mexico City in the early 1970s. The criteria considered included cost, capacity, access time to the airport, safety, social disruption, and noise pollution.



The problems in this chapter use the steps of multi-criteria decision making to make hard decisions. MCDM is a systematic approach to quantify an individual's preferences. Measures of interest are rescaled to numerical values on a 0–1 scale, with 0 representing the worst value of the measure and 1 representing the best. This allows the direct comparison of many diverse measures. In other words, with the right tool, it really is possible to compare apples to oranges! The result of this process is an evaluation of the alternatives in a rank order that reflects the decision makers' preferences.

For example, individuals, college sports teams, Master's degree programs, or even hospitals can be ranked in terms of their performance on many diverse measures. Another example is the Bowl Championship Series (BCS) in college football that attempts to identify the two best college football teams in the United States to play in a national championship bowl game. This process has reduced, but not eliminated, the annual end-of-year arguments as to which college should be crowned national champion.

Section 1.1: Choosing a Wireless Plan

Choosing a wireless plan is an important decision for many people. In fact, most teenagers own smart phones. When choosing a wireless plan, there are many factors to consider.

Q1. What factors would you consider if you were choosing a wireless plan?

In this chapter, you will develop a process for making important decisions, such as choosing a wireless plan, with many competing features. Before doing so, you will complete an opening activity.

1.1.1 Opening Activity

In this activity, you will make a decision about what wireless plan you would choose if you were considering a new plan. To do so, complete the following steps:

1. Make a list of possible wireless plans that you would consider using.
2. Collect data on each of these plans that you would find useful in making a decision.
3. Choose one of the plans based on your data.
4. Explain why you chose this plan over the others.

Q2. What possible issues do you foresee with using these steps to choose a wireless plan?

In the following sections, the steps of the MCDM process will be explained in the context of a high school student and her friend helping her parents to choose a wireless plan. Isabelle Nueva needs to help her mother and father decide on the best wireless plan to buy for their family. She and her friend, Angelo Franco, will use the MCDM process they learned in their math class to help her parents make this decision. Follow along with Isabelle as she and Angelo use the MCDM process to make this decision.

1.1.2 Identify Criteria and Measures

The first thing they do is identify the **criteria** of a wireless plan that were important to Isabelle's family. From discussions she had with her mother and father, Isabelle knew that the criteria that were important to them were cost, contract features, and phone service.

Q3. If you were choosing a wireless plan, what criteria would be important to you?

Isabelle and Angelo know that they need to find at least one way to **measure** each of the criteria. They decide to measure the Cost criterion using Monthly Charge, Monthly Access Fee, and Overage Fee. They decide to measure the Contract Features criterion using Number of GB of Data per Month, Rollover Data, and Contract Length. The measure of Phone Service is defined as Quality of Service. Each criterion and its measures are provided in Table 1.1.1.

Criteria	Measures
Cost	Monthly Charge
	Access Fee per Line
	Overage Fee (\$/GB)
Contract Features	Data Plan
	Rollover Data
	Contract Length
Phone Service	Quality of Service

Table 1.1.1: Criteria and measures for choosing a wireless plan

Q4. How would you measure each of your criteria?

The value of three measures—the Monthly Charge, Overage Fee, and Access Fee per Line—could be any numerical amount within a reasonable range. These are examples of **continuous measures**. That is, these measures can take on any numerical value within a range.

Isabelle and Angelo decide that the data they collected for the other three measures can be grouped into a finite number of categories. All of the plans they looked at before focusing on just three plans, had values that were multiples of 2.5 GB. They ranged from a low of 7.5 GB to a high of 15 GB. Thus, they decided to treat this as a **categorical measure** with only four possible values for Data Plan: 7.5, 10, 12.5, and 15 GB.

To obtain data on Quality of Service they decide to use ratings from a consumer magazine. The magazine considered dropped or disconnected calls, static and interference, and voice distortion to rate the quality of service. Isabelle and Angelo decide to only consider plans the magazine rated “Good”, “Very Good”, or “Excellent”. Therefore, this measure has three categories.

Another categorical measure is Contract Length—the shortest time a customer must remain with a particular plan to avoid paying a fee to cancel the service. Isabelle’s parents were concerned about being locked into a plan for a long period of time. The plans under consideration have only three different Contract Lengths (0, 1 year, and 2 years). All plans they investigate seem to use one of these. Thus, the Contract Length measure has three possible values. The categorical measures and their possible values are provided in Table 1.1.2.

Q5. Of the measures you listed in Q4, which are continuous and which should be treated as categorical?

Q6. Create a table similar to Table 1.1.2 for your categorical measures identified in the previous question. In order to do this, you will need to research possible wireless plans. What sort of research would you need to do?

Categorical Measure	Categorical Values (from best to worst)
Data Plan	15 GB 12.5 GB 10 GB 7.5 GB
Rollover Data	Yes No
Quality of Service	Excellent Very Good Good
Contract Length	0 1 year 2 years

Table 1.1.2: Categorical variables with categories and numeric values

1.1.3 Collect Data

Isabelle’s parents were considering three wireless plans: Trot, UST&T, and Horizon. Isabelle and Angelo collected the data they need to help her parents make their decision. The first data they collected were the basic monthly fees that appear in Table 1.1.3.

Angelo and Isabelle discussed the impact of the monthly access fee on the family’s cost. The Nueva family planned to initially sign up for four lines, one each for the parents and their two older teenagers. Angelo suggested that instead of two measures, these data should be combined into one measure, Total Monthly Charge. This is calculated by multiplying the per line fee by the number of lines and adding it to the base monthly fee. With this calculation, the monthly fee would be \$180 for Trot. The monthly fee for UST&T would be \$250. Lastly, Horizon would cost \$220 per month. However, Isabelle raised the possibility that her youngest brother who is in middle school might be given a fifth line. However, after some thought, they both agreed the cost of a fifth line should not be included in the decision analysis for now.

	Plan		
	Trot	UST&T	Horizon
Base Monthly Charge (\$)	100	130	20
Monthly Access Fee (\$/line)	20	30	50
Total Monthly Charge for Four Lines (\$)	180	250	220

Table 1.1.3: Wireless plans monthly cost

The other data they collected about the various plans are included in Table 1.1.4 alongside the monthly cost of four lines.

Plan	Trot	UST&T	Horizon
Total Monthly Charge (\$)	180	250	220
Overage Fee (\$/GB)	50	15	10
Contract Length	2 years	1 years	None
Data Plan (GB/month)	10	15	7.5
Rollover Data	No	Yes	No
Quality of Service	Excellent	Very Good	Good

Table 1.1.4: Isabelle and Angelo’s wireless plan data

Q7. Create a table similar to Table 1.1.4 for your wireless plan data.

1.1.4 Find the Range of Each Measure

Next, Isabelle and Angelo specify a range for each measure. They first specify the range for the two continuous measures (Total Monthly Charge and Overage Fee per GB). For each of these measures, they decide to use the range of the actual data they collected. That is, for Total Monthly Charge, the range was \$180 to \$250. The range Overage Fee was \$10 to \$50. For each of the categorical measures, Isabelle and Angelo simply list the two extreme values for each category. The **scale ranges** for each of Isabelle and Angelo’s measures are given in Table 1.1.5.

Measure	Scale range
Total Monthly Charge	\$180 to \$250
Overage Fee per GB	\$10 to \$50
Contract Length	0 to 2 years
Data Plan	7.5 GB to 15 GB
Rollover Data	Yes or No
Quality of Service	Good to Excellent

Table 1.1.5: Ranges of each measure

Q8. Specify the ranges for each of your measures, and create a table similar to Table 1.1.5.

1.1.5 Rescale Data on All Continuous Measures to a Common Unit

It would be difficult to compare the three plans using these raw data. For example, how would one compare a \$10 difference in the monthly service charge to a one-year difference in minimum contract length? In order to avoid such problems, operations researchers rescale the raw data of each measure to common unit values between zero and one. This creates a **common unit** that varies from zero to one for each measure. Zero always represents the worst value and one the best value for each measure.

For both of the continuous measures, Isabelle and Angelo use a **proportional scale** to assign a score to intermediate values. For example, the range for the Total Monthly Charge measure is \$180 to \$250. The smallest possible value here is the best option. Since the value one represents the best option, \$180 is converted to a common unit value of one. Similarly, the largest possible value of the monthly service charge is the worst option. Thus, \$250 converted to zero. That is,

\$180 → 1
 \$250 → 0

Next, Isabelle and Angelo convert the price of Horizon’s plan to a common unit value. They must decide what \$220 should be converted to when it is compared to the best and worst values for Total Monthly Charge. The graph in Figure 1.1.1 illustrates this.

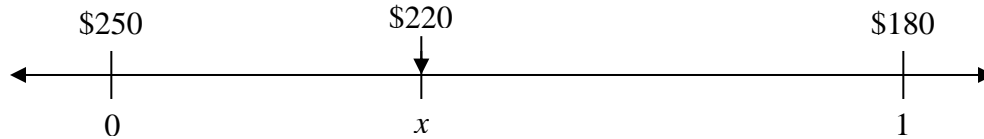


Figure 1.1.1: Determining the common unit values for the Total Monthly Charge measure

- Q9. What do you think \$220 should be converted to?
- Q10. Is \$220 closer to the best or the worst option?
- Q11. How far is \$220 from the best option? How far from the worst?

Isabelle and Angelo solve a proportion to arrive at the common unit value for the Total Monthly Charge of \$220. To find the common unit value for \$220 using proportions, Isabelle and Angelo write two equivalent fractions of the form $\frac{\text{part}}{\text{whole}}$. Figure 1.1.2 illustrates this.

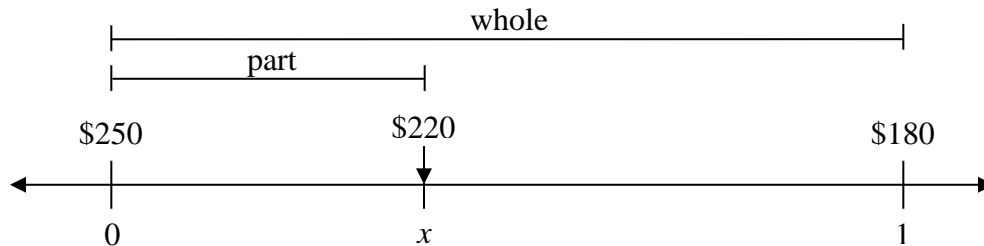


Figure 1.1.2: Determining the proportion to find the common unit values

In the first fraction, the “part” refers to the distance between \$250 and \$220, and the “whole” refers to the distance between \$250 and \$180. In the second fraction, the “part” refers to the distance between 0 and x , and the “whole” refers to the distance between 0 and 1. As can be seen in Figure 1.1.2, these two fractions are equivalent.

Isabelle and Angelo solve for the unknown in the equivalent fractions, using absolute value to find the distance between two values.

$$\frac{|220 - 250|}{|180 - 250|} = \frac{|x - 0|}{|1 - 0|}$$

$$\frac{30}{70} = \frac{x}{1}$$

$$0.42 = x$$

Therefore, the raw value \$220 is converted to the common unit value 0.42.

Notice, each time these equivalent fractions are developed, the fraction on the right will always be:

$$\frac{|x-0|}{|1-0|} = \frac{x}{1} = x$$

Therefore, there is no need to write the entire fraction. Simply x can be used instead.

Q12. What other ways could Isabelle and Angelo use to calculate the common unit value for \$220?

Q13. Find the common unit values for the Overage Fee per GB measure.

1.1.6 Rescale Each Categorical Measure to a Common Unit

For the four categorical measures, Isabelle and Angelo assign a common unit value of zero to the worst option and one to the best option. For the Rollover Data measure, the only possible values are “yes” and “no”. Yes was assigned a one, because is preferable; and no was assigned zero, because it is the worse value. When there was something between the best and worst values, Isabelle and Angelo discussed what to assign the intermediate values. With regard to Quality of Service, they simply assigned the one intermediate value, “very good,” a score of 0.5. They used analogous reasoning for the two intermediate values of the Data Plan measure. They assigned common units proportionately: 10 GB was 0.33 and 12.5 was 0.67. However, they knew that Isabelle’s parents really liked the idea of not being tied into a contract. A one-year contract was not much better than a two-year contract. They therefore assigned just 0.25 to a one-year contract. These conversions are summarized in Table 1.1.6.

Categorical Measure	Categorical Values	Common Units
Contract Length	0	1
	1 year	0.25
	2 years	0
Data Plan	15	1
	12.5	0.67
	10	0.33
	7.5	0
Rollover Data	Yes	1
	No	0
Quality of Service	Excellent	1
	Very Good	0.5
	Good	0

Table 1.1.6: Common unit values for the categorical measures

Isabelle and Angelo use the relationships developed above to convert the data for each plan into values between zero and one. The results of this conversion are presented in Table 1.1.7.

Plan	Trot	UST&T	Horizon
Total Monthly Charge (\$)	1	0	0.43
Overage Fee (\$/GB)	0	0.88	1
Contract Length	0	0.25	1
Data Plan (GB/month)	0.33	1	0
Rollover Data	0	1	0
Quality of Service	1	0.5	0
Total Points	2.33	3.63	2.43
Average Points	0.39	0.61	0.41

Table 1.1.7: Wireless plan data converted to a common unit

When Isabelle and Angelo looked at these results, they noticed that each plan received the top common unit value of one on two of the measures. They also noticed that each plan received at least one common unit value of zero. Therefore, it is not obvious to them which plan they should choose.

Q14. Based on the common unit values, which plan do you think Isabelle should recommend to her parents?

Angelo thinks they should use the total of all of the common units to get a total score for each plan. The totals are also listed in Table 1.1.7. Isabelle thinks it will be more meaningful to compute the average common unit scores for each plan. To do so, she divided the total score for each plan by six (the total number of measures and therefore the highest possible score). The averages she obtained are given in the bottom row of Table 1.1.7.

Q15. Do you think it makes more sense to use the sum or the average to make a decision?

Q16. Based on the total and average scores, which plan do you think Isabelle should recommend to her parents? Why?

Q17. What are some reasons why Isabelle may not recommend Horizon to her parents?

Q18. What are some reasons why Isabelle may think Trot would be a better choice for her parents?

Q19. What are some reasons why Isabelle may think UST&T would be a better choice for her parents?

Q20. Calculate the total scores and the average scores for each of your wireless plans.

- Based on these values, which plan would you choose?
- What are some reasons why these plans may not be the best choice for you?
- Was this plan what you expected to choose based on the opening activity? Why or why not?

Whether they use the sum or the average, Isabelle and Angelo realize that each plan has something in its favor. They wonder how to reach a decision. Then Isabelle remembers that her parents were really worried about the Total Monthly Charge, and not as worried about Contract Length. They decide that they need a system that does not treat all of the measures as equally important, as the sum and average do. They need a system that weights each measure according to how important it is to Isabelle’s parents.

1.1.6 Conduct an Interview to Calculate Weights

In order to learn how important each measure is to her parents, Isabelle and Angelo decide to interview them. They want to learn which measure Isabelle’s parents believe is most important to them. To do so, the parents will need to look closely at the most preferred value and least preferred value for each measure. Angelo and Isabelle explore with her parents how Isabelle’s parents would rank order the six measure ranges. Mr. and Mrs. Nueva decide that the difference between the highest and lowest monthly payments was most important to them. The difference between lowest and highest is \$70 per month; this is substantial. Therefore, they rank the Total Monthly Charge measure number one.

They knew their teenagers wanted to use their smart phones to download large files. They, therefore, rank Data Plan as the second most important measure. The Nueva’s rated the Quality of Service as the third most important measure. They might have ranked it higher if the scale included poor service. However, since the minimum was “good,” they were comfortable ranking it third most important. They really liked that Horizon offered a plan with no contract and therefore listed Contract Length as fourth. They were confident their children would strive to live within the monthly GB of data budget. However, they feared every once and a while they would lose track. In that case they could be shocked with a huge overage fee; they ranked Overage Fee fifth. They assumed their children would rarely have GBs of data to rollover into the next month. This measure was ranked last.

Table 1.1.8 shows their rank-ordering of the measures. For example, Total Monthly Charge is the most important measure to Isabelle’s parents and Rollover Data is the least important. This table also includes the least and the most preferred values for each measure.

Measure	Least Preferred Value	Most Preferred Value	Rank
Total Monthly Charge (\$)	250	180	1
Overage Fee (\$/GB)	50	10	5
Contract Length	2 years	0	4
Data Plan (GB/month)	7.5	15	2
Rollover Data	No	Yes	6
Quality of Service	Good	Excellent	3

Table 1.1.8: Rank-order of the measures according to Isabelle’s parents

Q21. Rank-order each of your measures.

Next, Isabelle and Angelo ask her parents to assign points to each measure to better capture the magnitude of the differences between two rankings. To make their decision-making model even more useful, they want a sense of how much more important one measure is than another. For example, if one measure is twice as important as another, then the assigned points should be twice as much for the higher ranked measure.

Isabelle and Angelo ask Mr. and Mrs. Nueva to assign 100 points to Total Monthly Charge, the measure they ranked number one. Then, they ask them to assign a number of points less than 100 to the second-ranked measure, Data Plan. In doing so, they ask Isabelle’s parents to pick a number that reflects how important Data Plan is compared to the Total Monthly Charge.

Mr. and Mrs. Nueva decide to assign 90 points to Family Data, because they know their children like to download large files. It is almost as important as the Total Monthly Charge. Quality of service was also important to them and only slightly less important than Data Plan. This was given 80 points. Although they liked not having a contract, it really was far less important than the first three measures. They assigned it 40 points, or half the weight of Quality of Service. The high overage fee was a risk they thought they could manage and gave it only 20 points. They did not think there was much value to their family of Rollover Data. They assigned it only 10 points. The Nueva’s preferences are summarized in Table 1.1.9.

Measure	Least Preferred Value	Most Preferred Value	Rank	Points
Total Monthly Charge (\$)	250	180	1	100
Overage Fee (\$/GB)	50	10	5	20
Contract Length	2 years	0	4	40
Data Plan (GB/month)	7.5	15	2	90
Rollover Data	No	Yes	6	10
Quality of Service	Good	Excellent	3	80

Table 1.1.9: Points assigned to each of the measures

Q22. Assign points to each of your measures, and create a table similar to Table 1.1.9.

Now, Isabelle and Angelo total all of the assigned points and obtain 340. Then, they divide the point assignment for each measure by that total. This number is the **weight** of that measure. For example, monthly charge was assigned 100 points. Thus, the weight of this measure is:

$$\frac{100}{340} = 0.29$$

One way of interpreting the weight of 0.29 for Total Monthly Charge is that 29% of the final decision will be based on this measure. The results of Isabelle and Angelo’s interview of her parents are summarized in Table 1.1.10.

Measure	Least Preferred Value	Most Preferred Value	Rank	Points	Weight
Total Monthly Charge (\$)	250	180	1	100	0.29
Overage Fee (\$/GB)	50	10	5	20	0.06
Contract Length	2 years	0	4	40	0.12
Family Data (GB/month)	7.5	15	2	90	0.26
Rollover Data	No	Yes	6	10	0.03
Quality of Service	Good	Excellent	3	80	0.24

Table 1.1.10: Calculated weight for each measure

- Q23. What measure has the largest weight? Which has the smallest?
- Q24. What is the ratio of the largest weight to the smallest weight?
- Q25. What should this ratio mean in the context of the decision?
- Q26. Assign points to each of your measures, and create a table similar to Table 1.1.10.

1.1.7 Calculate Total Scores

Now, Isabelle and Angelo calculate a **total score** for each plan. The total score is an example of a **weighted average**. They multiply each common unit value from Table 1.1.7 by the corresponding weight from Table 1.1.10. Then for each plan, they sum those six products together to get the total score. The data from these two tables are placed side-by-side in Table 1.1.11. The results of these computations are given in Table 1.1.12. Notice that this weighted average captures how important the various measures are to Isabelle's parents.

Measure	Weight	Trot	UST&T	Horizon
Total Monthly Charge (\$)	0.29	1	0	0.43
Overage Fee (\$/GB)	0.06	0	0.88	1
Contract Length	0.12	0	0.25	1
Family Data (GB/month)	0.26	0.33	1	0
Rollover Data	0.03	0	1	0
Quality of Service	0.24	1	0.5	0

Table 1.1.11: Measure weights and wireless plan scores

Measure	Weight	Trot	UST&T	Horizon
Total Monthly Charge (\$)	0.29	1×0.29 = 0.29	0×0.29 = 0	0.43×0.29 = 0.13
Overage Fee (\$/GB)	0.06	0×0.06 = 0	0.88×0.06 = 0.05	1×0.06 = 0.06
Contract Length	0.12	0×0.12 = 0	0.25×0.12 = 0.03	1×0.12 = 0.12
Family Data (GB/month)	0.26	0.33×0.26 = 0.09	1×0.26 = 0.26	0×0.26 = 0
Rollover Data	0.03	0×0.03 = 0	1×0.03 = 0.03	0×0.03 = 0
Quality of Service	0.24	1×0.24 = 0.24	0.5×0.26 = 0.13	0×0.26 = 0
Wireless Plan's Total Score		0.62	0.50	0.31

Table 1.1.12: A weighted total score is computed for each plan.

- Q27. Multiply the common unit values by the corresponding weights for each of your plans, and create a table similar to Table 1.1.12.
- Q28. Would everyone's score results lead to the same preferred choice? Explain.

1.1.8 Determine Strengths/Weaknesses and Make Final Decision

Trot is clearly the preferred plan. UST&T is a distant second. Isabelle and Angelo decide to closely examine the results. They clearly do not produce the same results as the sum or average methods.

- Q29. For which measures does Trot have a higher weighted score than UST&T? For which does UST&T outscore Trot?

When Isabelle and Angelo compare Trot with UST&T, they see that Trot had higher weighted scores for the first- and third-ranked measures, Total Monthly Cost and Quality of Service. UST&T scored higher on the other four measures. However, the magnitude of the difference for measures ranked four, five, and six was always small. In each case the difference was only 0.03 or less. These could not overcome the advantage Trot had on Total Monthly Cost, the highest ranked measure. Their weighting system did what it was supposed to do; it took into account Mr. and Mrs. Nueva's preferences. They decide to recommend the Trot plan to Isabelle's parents.

1.1.9 Alternative Trot Plan

Trot recently announced an alternative that comes with a larger Data Plan. This plan comes with 12.5 GB of data per month. It also costs \$15 a month more. The two plans are compared in Table 1.1.13.

Plan	Old Trot	New Trot
Total Monthly Charge (\$)	180	195
Overage Fee (\$/GB)	50	50
Contract Length	2 years	2 years
Data Plan (GB/month)	10	12.5
Rollover Data	No	No
Quality of Service	Excellent	Excellent

Table 1.1.13: Alternative Trot Plan

To compare the two plans, Isabelle and Angelo must first convert the new values to common units between zero and one. Then they will need to multiply the values by their corresponding weights.

- Q30. What is the common unit value for the monthly charge of \$195?
- Q31. What is the common unit value for the Data Plan, 12.5 GB per month?
- Q32. Should Isabelle and Angelo recommend to Mr. and Mrs. Nueva that they adopt the new Trot plan?

1.1.10 Summary

In this problem, Isabelle and Angelo wanted to help Isabelle's parents choose a wireless plan. They completed the following steps:

1. Identify criteria and measures
2. Collect data
3. Find the range of each measure
4. Rescale each measure to a common unit

After completing these steps, Isabelle and Angelo found the total score and the average score for each wireless plan. However, they noticed that these values treated all measures under consideration as being equally important. This was not a reasonable way to make a decision. They needed a way to weight some measures more than others, because Isabelle's parents were more concerned about the cost of the plan than anything else.

In order to take Mr. and Mrs. Nueva's preferences regarding a wireless plan into account, Isabelle and Angelo completed four additional steps:

5. Conduct an interview to rank order measures and assign points

6. Calculate the weight of each measure
7. Calculate a total score for each alternative
8. Interpret results

This eight-step process will be applied in the next two sections and in the homework problems to make slightly more complicated decisions. This process is also a life skill, because you may find it useful to help you make important decisions in your future.